

1. INTRODUCTION TO STATISTICS.

Singular Sense :

It is the science which deals with the collection analysis and Interpretation of numerical data is called statistics.

Defination :

According to Horace Savitt defined as "Statistics means aggregate of facts affected to a marked extent by multiplicity of cause, numerically Expressed, Estimated according to reasonable standard of accuracy, collected in a systematic manner for a pre-determined purpose and place in Relation to each other."

Objetives :

- To present facts in numerical form.
- To simplify, clarify and analyze, numerical data.
- To Bringout the Broad characteristic of a group in which the Individual members.
- To general, Economic and social conditions as a guide to administrate use.
- To Indicate business Trends and tendency.

function of statistics .

- classification of data
- comparison and simplify of data.
- relationship b/w facts
- movement & formulation of economic
- forecasting.

Limitations of statistics .

1. It doesn't study Quantitative phenomenon .
2. It doesn't study Individual data , it deals with aggregate of facts .
3. Statistical laws are not Exact laws .
4. It doesn't reveal the Entire Information .
5. It is liable to be misused .
6. Statistical conclusions are valid only on average base .

Collection of Data .

The process of collecting of data from survey or from Independent or network location via data capture .

Type of Collection of data .

1. primary data .
2. Secondary data .

→ primary data.

primary data is fresh data / first hand data collecting by the Investigator.

→ secondary data

The data which has been already collected by the Investigator.

classification of data.

→ univariate Data .

→ Bivariate Data .

→ multivariate Data .

1. univariate Data .

It is one kind of classification of data which consist of observation of facts in a single value or only one value of data.

2. Bivariate Data .

It means two different value of variable and its consistent of observations of data.

3. Multivariate Data .

It means two or more value of variable of data and finally results outcomes is single.

Tabulation.

which consist of Rows and Columns is called Tabulation.

Eg: material status wise.

- class wise
- sex wise.
- result wise.
- habit wise.
- colour wise

Blank table of Tabulation.

tabulation in the form of

Student class	Korhys Collg		Reva Collg		Total
	male	female	male	Female.	
A					
B					
Total:					

(1) prepare a tabulation form.

Town A - 56% male & 12 female coffee drinkers of female male non-coffee drinkers 10%.

Town B - 40% female are coffee drinkers 20% of males non-coffee drinkers, females are 50%.

Tabulation in the form of sex wise and habit wise.

cities \ Habit	male	Town A		Town B		Total (Total)
		Female	(Total)	Male	Female	
Coffee drinkers	46	12	58	30	40	70
Non-coffee drinkers	10	32	42	20	16	30
Total	56	44	100	50	50	100

(2) In 2020 total strength of 3 college ABC in a city is 3:1:2

Total strength of college A: 3000.

The proportion of Boys and girls in all college 1:3.

The faculty wise distribution of girls and Boys in the faculty of arts, science, commerce in Ratio 2:1:2

Respectively in all college.

Tabulate the above information.

Section	family wise	Boys			Total	Girls			Total	Total
		A	S	C		A	S	C		
				750	900	450	900	2250	3000	
A	300	150	300	450	300	150	300	750	1000	
B	100	50	100	250	100	600	1200	3000	4000	
C	400	300	400	1000	2400					
					1400	1100	2400	6000	8000	
Total	800	1100	800	2000						

3 In the sample study about food habit of Resident of a
Lannur.

where the following data were observed 55%
Residents were male, 85% vegetarian only 12% were non-
veg for female.

Tabulate the following.

Characteristics

sex - Male/female
food habit - veg/nonveg

Tabulation in the form of sex and food habit wise.

Sex	male	female	Total
food habit			
Veg	52	33	85
Non-veg	3	12	15
Total	55	45	100

In a sample study about food habits in two towns the data was obtained.

Town A - 50% were males 30% were non-veg, 18% non-veg males.

Town B - 45% were males 25% were non-veg 16% were male non-veg

Tabulate the above the following.

Food habit \ City	Town A			Town B		
	Male	Female	Total	Male	Female	Total
Veg	32	38	70	29	46	75
Non-veg	18	12	30	16	9	25
Total	50	50	100	45	55	100

The city of Kolari was divided into 3 geas administrative districts urban district Rural district

A survey of housing conditions were gathered.

There were 677100 Buildings in a city of which 176100 were in Rural district of the Building in urban district, of 406400 were in habitant 4500 were under construction.

In the administrative district, 4000 Buildings in the city were uninhabitat and 500 were under construction of the total 61600.

The total number of buildings in the City

under construction 62000 and uninhabited was 44500.

Tabulation in the form of area and housing condition.

areas ↓	Housing Con →	Inhabited	uninhabited	under construction
administrative		57100	4000	500
urban		406400	28500	4500
Rural		107000	12000	57000
Total		570600	44500	62000

In a state there were 30,00,000 out of these, 10,00,000 people lived in urban areas and the rest in Rural areas.

In urban area there were 70,000 male people, out of which 2,50,000 are literate in urban areas. 20,000 ladies were illiterate.

In Rural areas there were 15,00,000 male people out of which 50,000 were literate. In Rural areas illiterate were 30,00,000 ladies.

area	sex/ literacy	Male			Female			Total		
		l	ill	Total	l	ill	total	l	ill	T
Urban		2.5K	5	7.1akh	1	2.1akh	3	3.5	6.5	10.1
Rural		5K	10.2	15K	2	3.1akh	5	7	13	20.2
Total		7.5	14.5lak	22	3	8	8	10.5	19.5	30.2

Parts of a Table...

- Title:** It is very important of the table and the title should write or mention compulsory.
- Head Note:** It is one of the important note for the table.
- Stubs:** Stubs is maintained to the left side of the corner of a table.
- Caption:** It should be maintained in every table on the top of the table in horizontally.
- Body of the letter:** It is very important like a subject or matter should write in the middle of a table.
- Foot note:** In every parts of the table it will be the foot note, it cannot usage of this area

2. Measures of Central Tendency...

Average :

An average is a single figure which sums up the characteristics of a whole group of figures.

Type of Average.

1. Arithmetic Mean.
2. Median
3. Mode.

Arithmetic Mean:

It is defined as the value obtained by dividing the total values of all items in the series by their number.
It is denoted by \bar{X} .

Merits of Arithmetic Mean.

- It is Rigorously defined.
- It is Easy to calculate.
- It is Easy to understand.
- It is Very popular.
- It is Based on all the observations.
- It can be easily used for comparison.
- It is not affected by sampling fluctuations.

Direct Method - Mean.

$$\bar{X} = \frac{\sum X}{N}$$

Individual Series

Step deviation method:

$$\text{Direct method} = \left[\bar{x} = \frac{\sum fx}{N} \right]$$

$$\text{Shortcut method} = \left[\bar{x} = A + \frac{\sum fx}{N} \right]$$

$$\text{Step deviation method} = \left[\bar{x} = A + \frac{\sum fx}{N} \times CI \right]$$

Here, $\sum fx$ = summation of all observation.

N = Total number of items.

A = assumed mean.

CI = class interval.

Individual series of arithmetic mean.

(1) Calculate arithmetic mean from the following data.

x

2

3

5

4

6

7

3

$\Sigma x = 30$

$$\bar{x} = \frac{\Sigma x}{N}$$

$$= \frac{30}{7}$$

$$= 4.285$$

(8) Calculate mean from the following values

138, 138, 132, 135, 140, 148, 135, 139, 159, 161

\bar{x}

138

138

132

135

140

148

135

139

159

161

$\Sigma x = 1440$

$$\bar{x} = \frac{\Sigma x}{n}$$

$$= \frac{1440}{10}$$

$$\bar{x} = 144$$

(9) The monthly income of 10 family of a certain locality are given in ₹ as below. calculate mean.

Family	A	B	C	D	E	F	G	H	I	J
Income	850	700	840	750	800	800	420	5000	2300	1500

$$\bar{x} = \frac{\Sigma x}{n}$$

$$= \frac{11160}{10}$$

$$\bar{x} = 1116$$

(4.) Calculate Arithmetic mean of the weight of 10 students in a class.

SL. N	1	2	3	4	5	6	7	8	9	10
W(kg)	44	56	49	50	48	52	50	47	51	53

$$\bar{x} = \frac{\sum x}{n}$$

$$= \frac{470}{10}$$

$$\bar{x} = 47$$

Discrete series.

$$\bar{x} = \frac{\sum fx}{N}$$

(1.) Following are the marks obtained by students of a class in statistics. calculate Arithmetic mean.

Marks	35	40	45	50	55	60	65	70	75	80	85	90	95
No. of st	3	8	12	9	4	7	15	5	10	7	5	3	2

x	frequency.	xf
35	3	105
40	8	320
45	12	540
50	9	450
55	4	220
60	7	420
65	15	975
70	5	350
75	10	750
80	7	560
85	5	425
90	3	270
95	2	190
	$N=90$	$\sum fx = 5575$

$$\bar{x} = \frac{\sum fx}{N}$$

$$= \frac{5575}{90} = 61.94$$

$$\bar{x} = 61.94$$

2. The coins are tossed one thousand 24 times the theoretical frequency of 10 heads up to 0 heads are given below

No of heads	0	1	2	3	4	5	6	7	8	9	10
f	1	10	45	120	210	252	210	120	45	10	1

x	f	fx
0	1	0
1	10	10
2	45	90
3	120	360
4	210	840
5	282	1060
6	210	1260
7	120	840
8	45	360
9	10	90
10	1	10

$N = 1024$ $\sum fx = 5120$

$$\bar{x} = \frac{\sum fx}{N}$$

$$= \frac{5120}{1024}$$

$$\bar{x} = 5$$

Continuous series of Exclusive class Interval.

1. Calculate Arithmetic Mean from the following data.

production	10-20	20-30	30-40	40-50	50-60	60-70	70-80
No. of fac	5	4	7	12	10	8	4

Don't include in the deviation

CI	f	x(mu)	(fx)	$d = \frac{x-A}{c}$	fd
10-20	5	15	75	-3	-15
20-30	4	25	100	-2	-8
30-40	7	35	245	-1	-7
40-50	12	45	540	0	0
50-60	10	55	550	1	10
60-70	8	65	520	2	16
70-80	4	75	300	3	12
	$N=50$		$\sum fx = 2330$		$\sum fd = 8$

$$\bar{x} = \frac{\sum fx}{N}$$

$$= \frac{2330}{50}$$

$$\bar{x} = 46.6$$

$$\bar{x} = A + \frac{\sum fd}{N} \times CI$$

$$= 45 + \frac{8}{50} \times 10$$

$$= 45 + \frac{80}{50}$$

$$\bar{x} = 46.6$$

2. Calculate Arithmetic Mean from the following data.

Marks	0-10	10-20	20-30	30-40	40-50
No of st	3	5	2	4	1

C.I	f	x	fx
0-10	3	5	15
10-20	5	15	75
20-30	2	25	50
30-40	4	35	140
40-50	1	45	45

$N=15$ $\Sigma fx=325$

$$\bar{x} = \frac{\Sigma fx}{N}$$

$$= \frac{325}{15}$$

$$\bar{x} = 21.666$$

3. Calculate Mean.

C.I	f	x	fx
0-5	8	2.5	20
5-10	7	7.5	52.5
10-15	5	12.5	62.5
15-20	10	17.5	175
20-25	10	22.5	225
25-30	5	27.5	137.5
30-35	5	32.5	162.5

50 $\Sigma fx=835$

$$\bar{x} = \frac{\Sigma fx}{N}$$

$$= \frac{835}{50}$$

$$\bar{x} = 16.7$$

Inclusive.

(1) Calculate arithmetic mean for the following data.

Production	5-9	10-14	15-19	20-24	25-29	30-34	35-39
No of fac.	15	14	17	22	8	7	11

C I	freq	Mid(x)	fx
4.5-9.5	15	7	105
9.5-14.5	14	24	338 ¹⁶⁸
14.5-19.5	17	17	204 ²⁹⁸
19.5-24.5	22	17	484
24.5-29.5	8	27	216
30.5-34.5	7	32	224
34.5-39.5	11	37	407
	N=94		$\Sigma fx = 1893$

$$\bar{x} = \frac{\Sigma fx}{N}$$

$$\bar{x} = \frac{1893}{94}$$

$$\bar{x} = 20.138$$

2) Calculate Mean from the following data.

Profit	4-7	8-11	12-15	16-19	20-23	24-27	28-31	32-35	
No. of Com	6	10	18	30	15	12	10	6	
									36-39
									2

Class Interval	freq	x	fx
3.5-7.5	6	5.5	33
7.5-11.5	10	9.5	95
11.5-15.5	18	13.5	243
15.5-19.5	30	17.5	525
19.5-23.5	15	21.5	322.5
23.5-27.5	12	25.5	306
27.5-31.5	10	29.5	295
31.5-35.5	6	33.5	201
35.5-39.5	2	37.5	75
	<u>$N=109$</u>		<u>$\Sigma fx=2095.5$</u>

$$\bar{x} = \frac{\Sigma fx}{N}$$

$$= \frac{2095.5}{109}$$

$$\bar{x} = 19.22$$

3. Calculate Mean value.

Weight	71-75	76-80	81-85	86-90	91-95	96-100	101-105	106-110	
No. of Stu	3	10	15	18	25	19	14	9	
									111-115
									2

Class Interval	frequency	x	fx
70.5 - 75.5	3	136.5	219
75.5 - 80.5	10	156.5	780
80.5 - 85.5	15	83	1245
85.5 - 90.5	18	88	1584
90.5 - 95.5	25	93	2325
95.5 - 100.5	19	98	1862
100.5 - 105.5	14	103	1442
105.5 - 110.5	9	108	972
110.5 - 115.5	2	113	226
$N = 115$		$\Sigma fx = 10655$	

$$\bar{x} = \frac{\Sigma fx}{N}$$

$$= \frac{10655}{115}$$

$$\bar{x} = 92.652$$

Calculate the Mean Value

Marks	more 20	30	40	50	60	70	80	90	100
No of stu	10	18	25	32	43	61	67	85	100

class interval	frequency	x	fx
20-30	10	25	250
30-40	18	35	630
40-50	25	45	1125
50-60	32	55	1760
60-70	43	65	2795
70-80	61	75	4575
80-90	67	85	5695
90-100	85	95	8075
100-110	100	100	10000
	$N=100$		$\Sigma fx = 7080$

$$\bar{x} = \frac{\Sigma fx}{N}$$

$$= \frac{7080}{100}$$

$$\bar{x} = 70.80$$

Calculate Mean.

marks	above 20	30	40	50	60	70	80	90
No of H	100	95	87	62	43	25	13	2

Class Interval	f	%	fx	x	fx
20-30	100	100	500	25	125
30-40	95	58	475	35	280
40-50	87	825	696	45	1125
50-60	62	2519	1550	55	1045
60-70	43	1918	817	65	1170
70-80	25	1812	450	75	900
80-90	13	1211	156	85	935
90-100	2	112	22	95	190
		<u>N=100</u>			<u>Σfx=5770</u>

$$\bar{x} = \frac{\sum fx}{N}$$

$$= \frac{5770}{100}$$

$$= 57.70$$

Median.

Meaning: Median is the values of variables which divides the group into two equal parts & is called Median. It is denoted by "Med".

Merits of Median.

- It is rigidly defined.
- It is Easy to calculate.
- It is Easy to understand.

- * It is not affected by extreme values like arithmetic mean
- It can be located and graphically of the median.
- It can be used for qualitative studies.

Individual and Discrete Series of Median Formula.

Median is the size of $\frac{N+1}{2}$ term. — Absolute Median

Continuous Series of the Median Formula.

Median is the size of $n/2$ item.

$$\text{Median} = l + \frac{n/2 - cf}{f} \times CI$$

l = lower limit.

$n/2$ = median value.

cf = cumulative frequency.

f = frequency.

CI = class interval.

1. Calculate Median.

25, 25, 23, 40, 27, 25, 23, 25, 20.

15, 20, 23, 23, 25, 25, 25, 27, 40.

$$\text{Median} = \frac{N+1}{2}$$

$$= \frac{9+1}{2} = \frac{10}{2} = 5^{\text{th}}$$

∴ The median = 25

2. Calculate Median for the following.

10, 15, 14, 20, 28, 31, 40, 10, 8, 45, 41.

8, 10, 10, 14, 15, 20, 28, 31, 40, 41, 45.

$$\text{Median} = \frac{N+1}{2}$$

$$= \frac{11+1}{2}$$

$$= \frac{12}{2}$$

$$= 6^{\text{th}}$$

$$\therefore \text{Median} = 20$$

3. Calculate Median.

R.N. 1 2 3 4 5 6 7 8 9 10.

Marks 43 48 65 57 31 60 37 48 78 59.

R.N. Marks

1 31

2 37

3 43

4 48

5 48

6 57

7 59

8 60

9 65

10 78

$$\text{Median} = \frac{N+1}{2}$$

$$= \frac{10+1}{2} = \frac{11}{2} = 5.5^{\text{th}} \text{ term}$$

Between 5th 5th & 6th

$$\begin{aligned} &= \frac{45 + 57}{2} \\ &= \frac{102}{2} \\ &= 51 \end{aligned}$$

4. Discrete Series of Median.

Calculate the Median for the following data.

Size	4	6	8	10	12	14	16
freq.	2	4	5	3	2	1	4

Size	frequency	cf
4	2	2
6	4	2+4=6
8	5	6+5=11
10	3	11+3=14
12	2	14+2=16
14	1	16+1=17
16	4	17+4=21
N=21		

$$\text{Median} = \frac{N+1}{2}$$

$$= \frac{21+1}{2}$$

$$= \frac{22}{2} = 11$$

∴ Median is 11th term 8.

5. Calculate median from the following data.

Marks	50	40	30	20	10
freq.	10	40	20	12	16

Marks	freq	cf
50	10	10
40	40	50
30	20	70
20	12	82
10	16	98
	98	

$$\begin{aligned} \text{Median} &= \frac{N+1}{2} \\ &= \frac{98+1}{2} \\ &= \frac{99}{2} \\ &= 49.5 \end{aligned}$$

$$\text{Median} = 40$$

Continuous Series.

1. Marks	0-10	10-20	20-30	30-40	40-50	50-60	60-70
no.s.	5	8	7	12	28	20	10

C-I	freq	cf
0-10	5	5
10-20	8	13
20-30	7	20
30-40	12	32
40-50	28	60
50-60	20	80
60-70	10	90
	90.	

$$\text{Med} = \frac{N+1}{2} \cdot \frac{n}{2}$$

$$= \frac{90+1}{2} = \frac{91}{2} = 45$$

$$\text{Median} = l + \frac{n/2 - cf}{f} \times CI$$

$$= 40 + \left[\frac{45 - 32}{28} \right] \times 10$$

$$= 40 + \frac{13}{28} \times 10$$

$$= 40 + \frac{130}{28}$$

$$= 40 + 4.64$$

$$\text{Median} = 44.64$$

2 calculate median.

Wages.	5-10	10-15	15-20	20-25	25-30	30-35	35-40
No. of W	5	8	10	20	18	7	3

CI	freq	cf
5-10	5	5
10-15	9	13
15-20	10	23
20-25	20	43
25-30	18	61
30-35	7	68
35-40	3	71
	<u>71</u>	

$$\text{Median} = \frac{n}{2} = \frac{71}{2} = 35.5$$

$$\text{Median} = l + \left[\frac{\frac{n}{2} - cf}{f} \right] \times CI$$

$$= 20 + \frac{35.5 - 23}{20} \times 5$$

$$= 20 + \frac{12.5}{20} \times 5$$

$$= 20 + \frac{62.5}{20}$$

$$= 20 + 3.125$$

$$\text{Median} = 23.125$$

Mode.

mode means which is the highest value in the frequency.
(OR) in other words it means which item number of times repeated from the frequency is called mode.

It is denoted by 'Z'

Merits of mode.

- It is very easy to understand.
- It is easy to calculate.
- It is very popular.
- It is not extreme value of mean/median.
- It is graphical representation.

$$\text{Mode } Z = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times CI.$$

(1) Calculate

2, 5, 7, 13, 6, 8, 10,

mode is 13.

(2) Calculate

7, 10, 12, 13, 12, 12, 9, 6.

mode is 12.

(3) Calculate

15, 22, 35, 20, 10, 35, 35, 9, 7.

Mode is 35.

4 Calculate

CI	f	
0-10	3	$z = \frac{lt + f_1 - f_0}{2f_1 - f_0 - f_2} \times CI$
10-20	4 f_0	
<u>20-30</u>	<u>15</u> f_1	$= 20 + \frac{4 - 15}{2(15) - 4 - 10} \times 10$
30-40	10 f_2	
40-50	12	$= 20 + \frac{11}{30 - 4 - 10} \times 10$
		$= 20 + \frac{11}{26 - 10} \times 10$
		$= 20 + \frac{110}{16}$
		$= 20 + 6.875$
		Mode = 26.875

5. Calculate mode.

Marks	0-15	15-30	30-45	45-60	60-75	75-90	90-105
freq	8	6	14	25	10	4	2

CI	f	
0-15	8	
15-30	6	
30-45	14	f_0
<u>45-60</u>	<u>25</u>	f_1
60-75	10	f_2
75-90	4	
90-105	2	

$$Z = \frac{l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times CI}{}$$

$$= \frac{45 + \frac{25 - 14}{2(25) - 14 - 10} \times 15}{}$$

$$= \frac{45 + \frac{11}{50 - 14 - 10} \times 15}{}$$

$$= \frac{45 + \frac{11}{26} \times 15}{}$$

$$= \frac{45 + \frac{165}{26}}{}$$

$$= 45 + 6.346$$

Mode = 51.346

6. Calculate Mode...

CI	freq
10-19	6
20-29	4
30-39	11
40-49	20
50-59	7
60-69	8
70-79	2

Class I	freq.	CI	freq.
9.5-19.5	6	9.5-19.5	6
19.5-29.5	4	19.5-29.5	4
30.5-39.5	11	29.5-39.5	11 f ₀
40.5-49.5	20	39.5-49.5	20 f ₁
50.5-59.5	7	49.5-59.5	7 f ₂
59.5-69.5	3	59.5-69.5	3
69.5-79.5	2	69.5-79.5	2
79.5-89.5	1		
89.5-99.5			

$$Z = L + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times CI$$

$$= 39.5 + \frac{20 - 11}{2(20) - 11 - 7} \times 10$$

$$= 39.5 + \frac{9}{40 - 11 - 7} \times 10$$

$$= 39.5 + \frac{9}{22} \times 10$$

$$= 39.5 + \frac{90}{22}$$

$$= 39.5 + 4.090$$

$$\text{Mode} = 43.59$$

7. Calculate mode

CI	f
0-50	3
50-100	5
100-150	7
150-200	18
200-250	2
250-300	4
300-350	6

$$Z = 2 + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times CI$$

$$= 150 + \frac{18 - 7}{2(18) - 7 - 2} \times 50$$

$$= 150 + \frac{11}{36 - 7 - 2} \times 50$$

$$= 150 + \frac{550}{27}$$

$$= 150 + 20.37$$

$$\text{Mode} = \underline{\underline{170.37}}$$

8 Calculate mode less than.

X	less than 10	20	30	40	50	60	70	80
f	460	16	40	76	96	112	120	125

Ques.

x	f.	a/freq.
0-10	4	4
10-20	16	12
20-30	40	24 f_0
30-40	76	36 f_1
40-50	96	20 f_2
50-60	112	16
60-70	120	8
70-80	125	5

$$\text{mode} = L + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times CI$$

$$= 30 + \frac{36 - 24}{2(36) - 24 - 20} \times 10$$

$$= 30 + \frac{12}{28} \times 10$$

$$= 30 + \frac{120}{28}$$

$$= 30 + 4.285$$

$$\text{Mode} = 34.285$$

9. calculate Mode

X	f.	X	freq.
10	115	10-20	12
20	103	20-30	15
30	88	30-40	20 f ₀
40	68	40-50	25 f ₁
50	43	50-60	20 f ₂
60	23	60-70	10
70	13	70-80	10
80	3	80-90	3

$$\text{mode} = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times c$$

$$= 40 + \frac{25 - 20}{2(25) - 20 - 20} \times 10$$

$$= 40 + \frac{5}{50 - 20 - 20} \times 10$$

$$= 40 + \frac{50}{10}$$

$$\text{Mode} = 45$$

10. Calculate Mean, Median, mode.

CI	freq	x	fx	cf	
0-10	3	5	15	3	
10-20	7	15	105	10	f_0
20-30	15	25	375	25	f_1
30-40	5	35	175	30	f_2
40-50	10	45	450	40	
	40		1120		

$$\text{mean} = \frac{\sum fx}{N}$$

$$= \frac{1120}{40}$$

$$= 28$$

$$\text{median} = \frac{N+1}{2} = \frac{40+1}{2} = \frac{41}{2} = 20.5 \text{ (X)}$$

$$\text{median} = L + \frac{n/2 - cf}{f} \times CI$$

$$= 20 + \frac{20.5 - 10}{15} \times 10$$

$$= 20 + \frac{10.5}{15} \times 10$$

$$= 20 + \frac{105}{15}$$

$$\text{median} = 26.667$$

$$= 20 + 76.66$$

$$\begin{aligned} \text{mode} &= \frac{l + f_1 - f_0}{2f_1 - f_0 - f_2} \times CI \\ &= \frac{20 + 15 - 7}{2(15) - 7 - 5} \times 10 \\ &= \frac{20 + 8}{30 - 7 - 5} \times 10 \\ &= \frac{20 + 80}{18} \\ &= 20 + 4.44 \\ &= 24.44 \end{aligned}$$

11 Calculate mean, median, Mode.

CI	freq.	x	fx	cf
0-30	15	15	225	15
30-60	5	45	225	20
60-90	30	75	2250	50
90-120	25	105	2625	75
120-150	10	135	1350	85
150-180	20	165	3300	105
180-210	10	195	1950	115
	115		11925	

$$\begin{aligned} n/2 &= 115/2 \\ &= 57.5 \end{aligned}$$

$$\text{mean} = \frac{\sum fx}{N} = \frac{11925}{115} = 103.695$$

$$\text{median} = l + \frac{n/2 - cf}{f} \times CI$$

$$= 90 + \frac{57.5 - 50}{25} \times 30$$

$$= 90 + \frac{7.5}{25} \times 30$$

$$= 90 + \frac{112.5}{25}$$

$$= 90 + 9$$

$$= 99$$

$$\text{mode} = l + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times CI$$

$$= 60 + \frac{50 - 5}{2(30) - 5 - 25} \times 30$$

$$= 60 + \frac{25}{60 - 5 - 25} \times 30$$

$$= 60 + \frac{25}{30} \times 30$$

$$= 60 + 750$$

$$= 60 + 250$$

$$= 85$$

Empirical Method.

$$z = 3\text{median} - 2\text{mean}$$

1. $z = 40$, median = ? and mean = 25

$$z = 3\text{Median} - 2\text{mean}$$

$$40 = 3\text{Me} - 2\text{mean}(25)$$

$$40 = 3m - 50$$

$$40 + 50 = 3\text{median}$$

$$90 = 3\text{median}$$

$$\text{median} = \frac{90}{3}$$

$$\boxed{\text{median} = 30}$$

2. $z = ?$ Median = 10 Mean = 15

$$z = 3\text{Median} - 2\text{mean}$$

$$= 3(10) - 2(15)$$

$$= 30 - 30$$

$$\boxed{z = 0}$$

3. $z = 45$. Median = 25 Mean = ?

$$z = 3\text{Median} - 2\text{mean}$$

$$45 = 3(25) - 2\text{mean}$$

$$45 = 75 - 2\text{mean}$$

$$2\text{mean} = 75 - 45$$

$$2\text{mean} = 30$$

$$\text{Mean} = \frac{30}{2}$$

$$\boxed{\text{mean} = 15}$$

4. $Z = 25$, median = 10 mean = ?

$$Z = 3 \text{ median} - 2 \text{ mean}$$

$$25 = 3(10) - 2 \text{ mean}$$

$$25 = 30 - 2 \text{ mean}$$

$$2 \text{ mean} = 30 - 25$$

$$2 \text{ mean} = 5$$

$$\text{mean} = \frac{5}{2}$$

$$\boxed{\text{mean} = 2.5}$$

Standard deviation

Standard deviation is square root deviation from mean is called standard deviation.

It is denoted by ' σ '

$$\text{Individual series} = \sqrt{\frac{\sum d^2}{N}}$$

$$\text{Discrete series} = \sqrt{\frac{\sum f d^2}{N}}$$

$$\text{Continuous series} = \sqrt{\frac{\sum f d^2}{N} - \left(\frac{\sum f d}{N}\right)^2}$$

1.	X	$d = x - \bar{x} (33.9)$	d^2
	25	-8.9	79.21
	26	-7.9	62.41
	29	-6.9	47.61
	30	-3.9	15.21
	32	-1.9	3.61
	35	1.1	12.21
	38	4.1	16.81
	40	6.1	37.21
	42	8.1	65.61
	44	10.1	102.01
	$\Sigma x = 339$		$\Sigma d^2 = 430.89$

$$\sigma = \sqrt{\frac{\Sigma d^2}{N}}$$

$$= \sqrt{\frac{430.9}{339}}$$

$$= \sqrt{43.09} = 6.59$$

2. Calculate Standard deviation

wages 5 10 15 20 25 30 35 40 45 50

$$\bar{x} = \frac{\Sigma xc}{N}$$

$$= \frac{275}{10} = 27.5$$

x	$d = x - \bar{x}$	d^2
5	-22.5	506.25
10	-17.5	306.25
15	-12.5	156.25
20	-7.5	56.25
25	-2.5	6.25
30	2.5	6.25
35	7.5	56.25
40	12.5	156.25
45	17.5	306.25
50	22.5	506.25
<u>275</u>		<u>2062.5</u>

$$\sigma = \sqrt{\frac{\sum d^2}{N}}$$

$$= \sqrt{\frac{2062.5}{275}}$$

$$= 7.5$$

$$= 2.738$$

Calculate Standard Deviation.

Values . 15 20 25 30 35 40 45 50

x	$d = x - \bar{x}$	d^2
15	-15	225
20	-10	100
25	-5	25
30	0	0
35	5	25
40	10	100
45	15	225
<u>210</u>		<u>700</u>

$$\bar{x} = \frac{\sum x}{N}$$

$$= \frac{210}{7} = 30.$$

$$\sigma = \sqrt{\frac{\sum d^2}{N}}$$

$$= \sqrt{\frac{700}{7}}$$

$$= \sqrt{100}$$

$$\sigma = 10$$

Problems on Discrete Series.

1. Calculate standard deviation.

x	f	fx	$d = x - \bar{x}$	d^2	fd^2
2	2	4	-3.868	14.89	29.78
4	3	12	-1.86	3.45	10.35
6	5	30	0.14	0.014	0.095
8	4	32	2.14	4.57	18.28
10	1	10	4.14	17.13	17.63
	$N=15$	88			$\sum fd^2 = 75.63$

$$\bar{x} = \frac{\sum fx}{N}$$

$$= \frac{88}{15}$$

$$= 5.8666.$$

$$\sigma = \sqrt{\frac{\sum fd^2}{N}}$$

$$= \sqrt{\frac{75.635}{15}}$$

$$= \sqrt{5.04}$$

$$\sigma = 2.244$$

2. Calculate standard deviation.

x	f	fx	$d = x - \bar{x}$	d^2	fd^2
3	5	15	-6.6	43.56	217.8
6	10	60	-3.6	12.96	129.6
9	5	45	-0.6	0.36	1.8
12	12	144	2.4	5.76	69.12
15	8	120	5.4	29.16	233.28
	<u>40</u>	<u>384</u>			<u>651.6</u>

$$\bar{x} = \frac{\sum fx}{N}$$

$$= \frac{384}{40}$$

$$= 9.6$$

$$\sigma = \sqrt{\frac{\sum fd^2}{N}}$$

$$= \sqrt{\frac{651.6}{40}}$$

$$= \sqrt{16.29}$$

$$\sigma = 4.036$$

3. Calculate standard deviation of Variance is 625.

$$\sigma = \sqrt{\text{Variance}}$$

$$\sigma = \sqrt{625}$$

$$\sigma = 5$$

4. Calculate standard deviation of Variance is 121

$$\sigma = \sqrt{\text{Variance}}$$

$$\sigma = \sqrt{121}$$

$$\sigma = 11$$

5. Standard deviation of Variance is 169

$$\sigma = \sqrt{\text{Variance}}$$

$$\sigma = \sqrt{169}$$

$$\sigma = 13$$

6. Calculate Standard deviation.

Class Interval	freq.	mv(x)	$d = x - A/c$	fd	fd^2
0-10	10	5	-2	-20	40
10-20	15	15	-1	-15	15
20-30	5	25	0	0	0
30-40	3	35	1	3	3
40-50	2	45	2	4	8
	<u>35</u>			<u>-28</u>	<u>66</u>

$$\sigma = \sqrt{\frac{\sum fd^2}{N} - \left(\frac{\sum fd}{N}\right)^2} \times CI$$

$$= \sqrt{\frac{66}{35} - \left(\frac{-28}{35}\right)^2} \times 10$$

$$= \sqrt{\frac{66}{35} + \frac{784}{1225}} \times 10$$

$$= \sqrt{1.885 + 0.642} \times 10$$

$$= \sqrt{1.243} \times 10$$

$$= 1.114 \times 10$$

$\sigma = 11.14$

7. Calculate standard deviation.

Marks CI	freq.	x	$d = \frac{x-A}{c}$	fd	fd^2
10-20	14	15	-3	-42	126
20-30	16	25	-2	-32	64
30-40	10	35	-1	-10	10
40-50	15	45(A)	0	0	0
50-60	20	55	1	20	20
60-70	5	65	2	10	20
70-80	5	75	3	15	45
	$N=85$			-39	285

$$\sigma = \sqrt{\frac{\sum fd^2}{N} - \left(\frac{\sum fd}{N}\right)^2} \times CI$$

$$= \frac{285}{85} - \left(\frac{-39}{85}\right)^2 \times 10$$

$$= \frac{3.352 - 152}{7225} \times 10$$

$$= \sqrt{3.352 - 0.210} \times 10$$

$$= \sqrt{3.142} \times 10$$

$$= 1.772 \times 10$$

$$\sigma = 17.72$$

8. Calculate standard deviation

CI	freq.	x	$d = x - A/c$	fd	fd^2
0-50	25	250	-3	-75	225
50-100	15	75	-4	-60	60
100-150	10	125	-1	-10	10
150-200	17	175 (A)	0	0	0
200-250	13	225	1	13	13
250-300	10	275	2	20	40
300-350	20	325	3	60	180
	<u>N=110</u>			<u>-22</u>	<u>528</u>

$$\sigma = \sqrt{\frac{\sum fd^2}{N} - \left(\frac{\sum fd}{N}\right)^2 \times CI}$$

$$= \sqrt{\frac{528}{110} - \left(\frac{-22}{110}\right)^2 \times 250}$$

$$= \sqrt{4.8 - \frac{484}{12100} \times 250}$$

$$= \sqrt{4.8 - 0.04 \times 250}$$

$$= \sqrt{4.76 \times 250}$$

$$= 2.181 \times 250$$

$$\sigma = 54.52 \quad \times \quad \sigma = 109.05$$

(Difference b/w two values of the Variable)

3. Correlation and Regression Analysis..

Correlation.

The difference b/w the two values of variables is called correlation

Types of correlation

- 1 Perfect positive correlation = +1
- 2 Perfect negative correlation = -1
- 3 Moderate degree correlation = 0.60 to 0.75
- 4 Lower degree correlation = 0.30 to 0.60.
- 5 Very low degree correlation = 0.01 to 0.30.

Methods of correlation.

- 1 Scatter diagram.
- 2 Karl Pearson's ^{coefficient} correlation
- 3 Spearman's rank correlation

$$\text{Karl Pearson's } r = \frac{\sum xy}{\sqrt{\sum x^2 \times \sum y^2}}$$

$$b_{xy} = \sqrt{\sum x^2}$$

$$b_{yx} = \sqrt{\sum y^2}$$

$$(r = \sqrt{b_{xy} \times b_{yx}})$$

1. The following data relate to the age of Employees and the number of days they were reported sick in a month.

(x)	(y)					
Age	Sick days	$x = x - \bar{x} (46)$	x^2	$y = y - \bar{y} (4)$	y^2	xy
30	1	-16	256	-3	9	48
32	0	-14	196	-4	16	56
35	2	-11	121	-2	4	22
40	5	-6	36	1	1	6
48	2	2	4	-2	4	4
50	4	4	16	0	0	0
52	6	6	36	2	4	12
55	5	9	81	1	1	9
57	7	11	121	3	9	33
61	8					
<u>460</u>	<u>40</u>	15	225	4	16	60
			1092		64	230

Calculate Karl Pearson's correlation?

$$\bar{x} = \frac{\sum x}{N} = \frac{460}{10} = 46$$

$$\bar{y} = \frac{\sum y}{N} = \frac{40}{10} = 4$$

$$r = \frac{\sum xy}{\sqrt{\sum x^2 \times \sum y^2}}$$

$$= \frac{230}{\sqrt{1092 \times 64}}$$

$$= \frac{230}{264.36}$$

$$r = 0.87$$

2. Calculate Karl Pearson's coefficient.

Economics(x) -Acctg(y)

48 45

35 20

17 40

23 25

47 45

x	y	$x - \bar{x}$	$y - \bar{y}$	x^2	y^2	xy
48	45	14	10	196	100	140
35	20	1	-15	1	225	-15
17	40	-17	5	289	25	-85
23	25	-11	-10	121	100	110
47	45	13	10	169	100	130
<u>170</u>	<u>175</u>			<u>776</u>	<u>550</u>	<u>280</u>

$$\bar{x} = \frac{\sum x}{N} = \frac{170}{5} = 34$$

$$r = \frac{\sum xy}{\sqrt{\sum x^2 \times \sum y^2}}$$

$$\bar{y} = \frac{\sum y}{N} = \frac{175}{5} = 35$$

$$= \frac{280}{\sqrt{776 \times 550}}$$

$$= \frac{280}{\sqrt{426800}}$$

low degree.

$$= \frac{280}{653.29} = 0.428$$

3 Compute Kaul Pearson's Coefficient of Correlation and Probable Error from the following data.

Acc(x)	Eng(y)	$x - \bar{x}$	x^2	$y - \bar{y}$	y^2	xy
77	35	28	784	-10	100	-280
54	58	5	25	13	169	65
27	60	-22	484	15	225	-330
52	40	3	9	-5	25	-15
14	50	-35	1224	5	25	-175
35	40	-14	196	-5	25	70
90	35	41	1681	-10	100	-410
25	56	24	576	11	121	-264
56	34	7	49	-11	121	-77
60	42	11	121	-3	9	-33
<u>490</u>	<u>450</u>		<u>5150</u>		<u>920</u>	<u>+135</u>
						<u>-1584</u>
						<u>-1449</u>

$$\bar{x} = \frac{\sum x}{N}$$

$$\bar{y} = \frac{\sum y}{N}$$

$$= \frac{490}{10} = \frac{450}{10}$$

$$r = \frac{\sum xy}{\sqrt{\sum x^2 * \sum y^2}}$$

$$= \frac{-1449}{\sqrt{5150 * 920}}$$

$$\boxed{\bar{x} = 49} \quad \boxed{\bar{y} = 45}$$

PE (probable error)

$$= 0.6745 \times \frac{1-r^2}{\sqrt{N}}$$

$$= \frac{-1449}{\sqrt{4738000}}$$

$$0.6745 \times \frac{1-(0.66)^2}{\sqrt{10}}$$

$$= \frac{-1449}{2176.69}$$

$$0.6745 \times \frac{1-0.4356}{3.162}$$

$$\boxed{r = -0.665} \text{ Negative}$$

$$= \frac{0.6745 \times 0.5644}{3.162}$$

$$= 0.6745 \times 0.178$$

$$\boxed{PE = 0.12039}$$

4. Calculate Karl Pearson's coefficient of correlation and PE.

A(x)	B(y)	$x - \bar{x}$	$y - \bar{y}$	x^2	y^2	xy
10	9	0	1	0	1	0
6	4	-4	-4	16	16	16
9	6	-1	-2	1	4	2
10	9	0	1	0	1	0
12	11	2	3	4	9	6
13	13	3	5	9	25	15
11	8	1	0	1	0	0
9	4	-1	-4	1	16	4
<u>80</u>	<u>64</u>			<u>32</u>	<u>72</u>	<u>43</u>

$$\bar{x} = \frac{\sum x}{N}$$

$$= \frac{80}{8}$$

$$\bar{y} = \frac{\sum y}{N}$$

$$= \frac{64}{8}$$

$$\boxed{\bar{x} = 10} \quad \boxed{\bar{y} = 8}$$

$$r = \frac{\sum xy}{\sqrt{\sum x^2 \times \sum y^2}}$$

$$= \frac{43}{\sqrt{32 \times 72}}$$

$$= \frac{43}{\sqrt{2304}} = \frac{43}{48} = 0.895$$

(Very low)

$$PE = \frac{0.6745 \times (1 - r^2)}{\sqrt{N}}$$

$$= \frac{0.6745 \times (1 - (0.89)^2)}{\sqrt{8}}$$

$$= \frac{0.6745 \times (1 - 0.7921)}{2.82}$$

$$= \frac{0.6745 \times 0.2079}{2.82}$$

$$= 0.6745 \times 0.071$$

PE = 0.0478

5 Calculate Karl Pearson's coefficient of correlation and PE

Height (x)	Weight (y)	x - \bar{x}	y - \bar{y}	x ²	y ²	xy
45	35	-16	-29	256	841	464
70	90	9	26	81	676	234
65	70	4	6	16	36	24
30	40	-31	-24	961	576	744
90	95	29	31	841	961	899
40	40	-21	-24	441	576	504
50	60	-11	-4	121	16	44
75	80	14	16	196	256	224
85	80	24	16	576	256	384
60	50	-1	-14	1	196	14
610	640			3496	4390	3535

$$\bar{x} = \frac{\sum x}{N}$$

$$= \frac{610}{10}$$

$$\bar{x} = 61$$

$$\bar{y} = \frac{\sum y}{N}$$

$$= \frac{640}{10}$$

$$\bar{y} = 64$$

$$r = \frac{\sum xy}{\sqrt{\sum x^2 \times \sum y^2}}$$

$$= \frac{3535}{\sqrt{3490 \times 4390}}$$

$$= \frac{3535}{\sqrt{15321100}}$$

$$= \frac{3535}{3914.21}$$

$$= 0.903$$

$$3914.21$$

$r = 0.903$ High degree

$$PE = 0.6745 \times \frac{1-r^2}{\sqrt{N}}$$

$$= 0.6745 \times \frac{1-(0.903)^2}{\sqrt{10}}$$

$$= 0.6745 \times \frac{1-0.8154}{3.162}$$

$$= 0.6745 \times \frac{0.1846}{3.162}$$

$$= 0.6745 \times 0.0583$$

$$PE = 0.0393$$

Spearman's Rank Co-relation..

$$r_s = 1 - \frac{6 \sum d^2}{N^3 - N}$$

If. Any common rank in the values .

$$r_s = 1 - \frac{6 \left\{ \sum d^2 + \frac{m^2 - m}{12} + \frac{m^3 - m}{12} + \dots \right\}}{N^3 - N}$$

Ranker are participated in beauty, voice and playing contest..

1. From the following data calculate co-efficient of Rank Correlation b/w X and Y.

X	36	58	20	65	42	33	44	50	15	60
Y	50	35	70	25	58	75	60	45	80	38

X	Y	R _X	R _Y	d = R _X - R _Y	d ²
36	50	7	6	1	1
56	35	3	9	-6	36
20	70	9	3	6	36
65	25	1	10	-9	81
42	58	6	5	1	1
33	75	8	2	6	36
44	60	5	4	1	1
50	45	4	7	-3	9
15	80	10	1	9	81
60	38	2	8	-6	36
					318

$$r_s = \frac{1 - 6 \sum d^2}{N^3 - N}$$

$$= \frac{1 - 6(318)}{10^3 - 10}$$

$$= \frac{1 - 1908}{990}$$

$$= 1 - 1.927$$

$$r_s = 0.927$$

2. From the following data, calculate the Co-efficient of Rank Correlation

X	Y	R _X	R _Y	d = R _X - R _Y	d ²
80	123	4	4	0	0
91	135	2	2	0	0
99	154	1	1	0	0
71	110	5	6	-1	1
61	105	7	8	-1	1
81	134	3	3	0	0
70	121	6	5	1	1
59	106	8	7	1	1
					$\sum d^2 = 4$

$$r_s = \frac{1 - 6 \sum d^2}{N^3 - N}$$

$$= \frac{1 - 6(4)}{8^3 - 8} = \frac{1 - 24}{504}$$

$$= 1 - 0.047$$

$$\boxed{r_s = 0.953}$$

3 Calculate Spearman's Rank Co-relation.

X	Y	R _X	R _Y	d = R _X - R _Y	d ²
60	73	1	1	0	0
34	32	8	7	1	1
40	34	7	5	2	4
50	40	2	3	-1	1
45	45	3	2	1	1
41	33	6	6	0	0
22	12	9	9	0	0
43	30	4	8	-4	16
42	36	5	4	1	1
					24

$$r_s = 1 - \frac{6 \sum d^2}{N^3 - N}$$

$$= 1 - \frac{6(24)}{9^3 - 9}$$

$$= 1 - \frac{144}{720}$$

$$= 1 - 0.2$$

$$\boxed{r_s = 0.8}$$

4 calculate Spearman's correlation.

J_1	J_2	R_1	R_2	$d = R_1 - R_2$	d^2
45	35	9	9	0	0
50	41	6	4	2	4
56	40	3	5.5	-2.5	6.25
75	35	1	9	8	64
50	35	6	9	3	9
46	60	8	1	7	49
40	40	10	5.5	4.5	20.25
50	38	6	7	-1	1
52	42	4	3	1	1
60	50	2	2	0	0

$$\sum d^2 = 154.50$$

$$50 = \frac{5+6+7}{3} = \frac{18}{3} = 6$$

$$r_s = 1 - \frac{\sum d^2}{N^3 - N}$$

$$40 = \frac{5+6}{2} = \frac{11}{2} = 5.5$$

$$35 = \frac{8+9+10}{3} = \frac{27}{3} = 9$$

$$r_s = 1 - \frac{6 \left\{ \frac{\sum d^2}{12} + \frac{m^3 - m}{12} + \frac{m^3 - m}{12} + \frac{m^3 - m}{12} \right\}}{N^3 - N}$$

$$= 1 - \frac{6 \left\{ \frac{154.5}{12} + \frac{3^3 - 3}{12} + \frac{2^3 - 2}{12} + \frac{3^3 - 3}{12} \right\}}{10^3 - 10}$$

$$= 1 - \frac{6 \{154 \cdot 50 + 2 + 0 \cdot 5 + 2\}}{990}$$

$$= 1 - \frac{6 \times 159}{990}$$

$$= 1 - \frac{954}{990}$$

$$= 1 - 0.963$$

$$\boxed{r_s = 0.036}$$

5. Calculate Rank Correlation

X	Y	R ₁	R ₂	d = R ₁ - R ₂	d ²
25	30	2	3.5	-1.5	2.25
18	35	7	1	6	49
20	34	6	2	4	16
24	20	3	8	-5	25
20	30	6	3.5	2.5	6.25
20	24	6	6	0	0
30	24	1	6	-5	25
21	24	4	6	-2	4
					127.5

$$20 = \frac{5+6+7}{3} = \frac{18}{3} = 6$$

$$24 = \frac{5+6+7}{3} = \frac{18}{3} = 6$$

$$30 = \frac{3+4}{2} = \frac{7}{2} = 3.5$$

$$r_s = 1 - \frac{6 \left\{ \sum d^2 + \frac{m^3 - m}{12} + \frac{m^3 - m}{12} + \frac{m^3 - m}{12} \right\}}{N^3 - N}$$

$$= 1 - \frac{6 \left\{ 127.5 + \frac{3^3 - 3}{12} + \frac{2^3 - 2}{12} + \frac{3^3 - 3}{12} \right\}}{8^3 - 8}$$

$$= 1 - \frac{6 \left\{ 127.5 + 2 + 0.5 + 2 \right\}}{504}$$

$$= 1 - \frac{6 \times 132}{504}$$

$$= 1 - \frac{792}{504} = 1 - 1.57$$

$$\boxed{r_s = -0.57}$$

$J_3 = J_1$

$H = 2$

$S = 3$

$B = 2$

$F = 2$

6 Calculate rank correlation B/w 3 judges of Beauty competition. which pair is nearest.

J_1	J_2	J_3	$d = J_1 - J_2$	d^2	Rank b/w $J_1 = J_2$	$d = J_2 - J_3$	d^2
4	5	8	-1	1	J_1 $4=2$ $5=3$ J_2 $4=3$ $3=2$	-3	9
3	4	7	-1	1		-3	9
1	4	5	-3	9		-1	1
2	3	6	-1	1		-3	9
5	3	7	2	4	-4	16	
5	2	8	3	9	-6	36	
6	1	3	5	25	-2	4	
7	6	2	1	1	4	16	
5	7	1	-2	4	6	36	
4	9	4	-5	25	5	25	
8	8	9	0	0	-1	1	
9	4	10	5	25	-6	36	
				$\Sigma d^2 = 105$			198

$$r_s = 1 - 6 \left\{ \frac{\Sigma d^2 + \frac{m^3 - m}{12} + \frac{m^3 - m}{12} + \frac{m^3 - m}{12} + \frac{m^3 - m}{12} \right\} \quad J_2 = J_3$$

$$= 1 - 6 \left\{ \frac{105 + \frac{2^3 - 2}{12} + \frac{3^3 - 3}{12} + \frac{3^3 - 3}{12} + \frac{2^3 - 2}{12} \right\} \quad \begin{matrix} 4=3 \\ 3=2 \\ 8=2 \\ 7=2 \end{matrix}$$

$$= 1 - 6 \left\{ \frac{105 + 0.5 + 2 + 2 + 0.5}{1716} \right\}$$

$$= \frac{1 - 6 \times 110}{1716} = \frac{1 - 660}{1716}$$

$$= 1 - 0.384 \quad \boxed{r_s = 0.616}$$

$J_2 - J_3$

$$\gamma_3 = \frac{1-6 \left\{ 2 \cdot 4^2 + \frac{m^3-m}{12} + \frac{m^3-m}{12} + \frac{m^3-m}{12} + \frac{m^3-m}{12} \right\}}{N^3 - N}$$

$$= \frac{1-6 \left\{ 198 + \frac{3^3-3}{12} + \frac{2^3-2}{12} + \frac{2^3-2}{12} + \frac{2^3-2}{12} \right\}}{12^3 - 12}$$

$$= \frac{1-6 \left\{ 198 + 2 + 0.5 + 0.5 + 0.5 \right\}}{1716}$$

$$= \frac{1-6 \left\{ \frac{201.5}{1716} \right\}}$$

$$= 1 - 6 \times 0.117$$

$$= 1 - 0.704$$

$$\boxed{\gamma_3 = 0.295}$$

$J_3 - J_1$

$$\gamma = \frac{1-6 \left\{ 127 + \frac{2^3-2}{12} + \frac{3^3-3}{12} + \frac{2^3-2}{12} + \frac{2^3-2}{12} \right\}}{12^3 - 12}$$

$$= \frac{1-6 \left\{ 129 + 0.5 + 2 + 0.5 + 0.5 \right\}}{1716}$$

$$= \frac{1-6 \left\{ \frac{132.5}{1716} \right\}}$$

$$= 1 - \frac{6 \times 130.5}{1716}$$

$$= 1 - \frac{783}{1716}$$

$$= 1 - 0.4563$$

$$y_s = 0.5437$$

Regression analysis.

Regression is measure of the variable of two or more values in form of another is called Regression.

Regression is returning back / stepping back to the average value of two or more variable of value.

No rank, then apply the formula.

Reg x on y

Reg y on x

$$x - \bar{x} = \frac{\sum xy}{\sum y^2} (y - \bar{y})$$

$$y - \bar{y} = \frac{\sum xy}{\sum x^2} (x - \bar{x})$$

$$x - \bar{x} = r \frac{\sigma_x}{\sigma_y} (y - \bar{y})$$

$$y - \bar{y} = r \frac{\sigma_y}{\sigma_x} (x - \bar{x})$$

Suppose, If there is rank

Q1 From the following data obtain the 2 equations.

x	y	$x - \bar{x}$	$y - \bar{y}$	x^2	y^2	xy
6	9	0	1	0	1	0
2	11	-4	3	16	9	-12
10	5	4	-3	16	9	-12
4	8	-2	0	4	0	0
8	7	2	-1	4	1	-2
$\Sigma x = 30$	$\Sigma y = 40$			40	20	-26

$$\bar{x} = \frac{\Sigma x}{n} = \frac{30}{5} = 6$$

Reg sq x on y.

$$\bar{y} = \frac{\Sigma y}{n} = \frac{40}{5} = 8$$

$$x - \bar{x} = \frac{\Sigma xy}{\Sigma y^2} (y - \bar{y})$$

$$x - 6 = \frac{-26}{20} (y - 8)$$

Reg sq on y and x.

$$x - 6 = -1.3y + 10.4$$

$$y - \bar{y} = \frac{\Sigma yx}{\Sigma x^2} (x - \bar{x})$$

$$x = -1.3y + 10.4 + 6$$

$$\boxed{x = -1.3y + 16.4}$$

$$y - 8 = \frac{-26}{40} (x - 6)$$

$$y - 8 = -0.65(x - 6)$$

$$y - 8 = -0.65x + 3.9$$

$$y = -0.65x + 3.9 + 8$$

$$\boxed{y = -0.65x + 11.9}$$

Q2. From the following data obtain the Regression and also calculate the average value of Y when x = 9.

X	Y	$x = x - \bar{x}$	$y = y - \bar{y}$	x^2	y^2	xy
3	3	-1.5	-1	2.25	1	1.5
6	2	1.5	-2	2.25	4	3
5	3	0.5	-1	0.25	1	-0.5
4	5	-0.5	1	0.25	1	-0.5
7	3	2.5	-1	6.25	1	-2.5
2	6	-2.5	2	6.25	4	-5
8	6	3.5	2	12.25	4	7
1	4	-3.5	0	12.25	0	0
36	32			42	16	11.5
						-8.5
						3

$$\bar{x} = \frac{\sum x}{n} = \frac{36}{8} = 4.5$$

Reg sq x on y.

$$x - \bar{x} = \frac{\sum xy}{\sum y^2} (y - \bar{y})$$

$$\bar{y} = \frac{\sum y}{n} = \frac{32}{8} = 4$$

$$x - 4.5 = \frac{3}{16} (y - 4)$$

Reg sq y on x.

$$y - \bar{y} = \frac{\sum yx}{\sum x^2} (x - \bar{x})$$

$$x - 4.5 = 0.1875y - 0.75$$

$$x = 0.1875y - 0.75 + 4.5$$

$$x = 0.1875y + 3.75$$

$$y - 4 = \frac{3}{42} (x - 4.5)$$

$$x = 9$$

$$y - 4 = 0.071x - 0.3195$$

$$y = 0.071x + 4.3195$$

$$= 0.071(9) + 4.3195$$

$$y = 0.071x + 4.3195 + 4$$

$$y = 0.639 + 4.3195$$

$$y = 0.071x + 4.3195$$

3. The following data relate to the age of husband/wife obtain 2 Regression Equations. The most likely to the age of husband when age of wife is 25 years.

husband (x)	wife (y)	$x = x - \bar{x}$	$y = y - \bar{y}$	x^2	y^2	xy
25	20	-11	-9	121	81	99
28	26	-8	-3	64	9	24
30	29	-6	0	36	0	0
32	30	-4	1	16	1	-4
35	25	-1	-4	1	16	-4
36	18	0	-11	0	121	-60
38	26	2	-3	4	9	-6
39	35	3	6	9	36	18
42	35	6	6	36	36	36
55	46	19	17	361	289	323
360	290			648	598	-10
						+504
						494

$$\bar{x} = \frac{\sum x}{n} = \frac{360}{10} = 36$$

$$\bar{y} = \frac{\sum y}{n} = \frac{290}{10} = 29$$

Reg eqn y on x

Reg eqn x on y:

$$y - \bar{y} = \frac{\sum xy}{\sum x^2} (x - \bar{x})$$

$$x - \bar{x} = \frac{\sum xy}{\sum y^2} (y - \bar{y})$$

$$y - 29 = \frac{494}{648} (x - 36)$$

$$x - 36 = \frac{494}{598} (y - 29)$$

$$y - 29 = 0.762x - 27.432$$

$$x - 36 = 0.826y - 23.954$$

$$y = 0.762x - 27.432 + 29$$

$$x = 0.826y - 23.954 + 36$$

$$y = 0.762x + 1.568$$

$$x = 0.826y + 12.046$$

$$x = 0.8264 + 12.046$$

$$x = 0.826(25) + 12.046$$

$$x = 20.65 + 12.046$$

$$\boxed{x = 32.696} \Rightarrow 33.$$

If they give rank.

(i)	x	y
	20	25 = Mean
	(2) 4	(2) 9 = Variance

Coefficient of correlation = 0.75 (r)

Req eq x on y

Req eq y on x.

$$x - \bar{x} = r \frac{\sigma_x}{\sigma_y} (y - \bar{y})$$

$$y - \bar{y} = r \frac{\sigma_y}{\sigma_x} (x - \bar{x})$$

$$x - 20 = 0.75 \frac{2}{3} (y - 25)$$

$$y - 25 = 0.75 \frac{3}{2} (x - 20)$$

$$x - 20 = 0.54 - 12.5$$

$$x = 0.54 - 12.5 + 20$$

$$\boxed{x = 0.54 + 7.5}$$

$$y - 25 = 1.125x - 22.5$$

$$y = 1.125x - 22.5 + 25$$

$$\boxed{y = 1.125x + 2.5}$$

(02) give the following.

$$\bar{x} = 65 \quad \bar{y} = 67.$$

$$s_y = 3.5 \quad s_x = 25 \text{ (standard deviation)}$$

$$r = 0.8.$$

Obtain regression sq, Estimate of x when $y = 70$.

y when $x = 58$.

Reg x on y

$$x - \bar{x} = r \frac{s_x}{s_y} (y - \bar{y})$$

$$x - 65 = 0.8 \frac{25}{3.5} (y - 67)$$

$$x - 65 = 5.714 (y - 67)$$

$$x - 65 = 5.714y - 382.83$$

$$x = 5.714y - 382.83 + 65$$

$$x = 5.714y - 317.83$$

Reg y on x

$$y - \bar{y} = r \frac{s_y}{s_x} (x - \bar{x})$$

$$y - 67 = 0.8 \frac{3.5}{25} (x - 65)$$

$$y - 67 = 0.112 (x - 65)$$

$$y - 67 = 0.112x - 7.28$$

$$y = 0.112x - 7.28 + 67$$

$$y = 0.112x + 59.72$$

$$x = 5.714y - 317.83$$

$$x = 5.714(70) - 317.83$$

$$= 399.98 - 317.83$$

$$x = 82.15$$

$$y = 0.112x + 59.72$$

$$= 0.112(58) + 59.72$$

$$= 6.496 + 59.72$$

$$y = 66.216$$

(c) Find the likely sales when advertisement Expenditure is ₹ 25 Cr

	Advertisement	Sales
mean	20	120
S.D	5	25
corr	+0.8	

Reg x on y.

Reg y on x.

$$x - \bar{x} = r \frac{\sigma_x}{\sigma_y} (y - \bar{y})$$

$$y - \bar{y} = r \frac{\sigma_y}{\sigma_x} (x - \bar{x})$$

$$x - 20 = 0.8 \frac{5}{25} (y - 120)$$

$$y - 120 = 0.8 \frac{25}{5} (x - 20)$$

$$x - 20 = 0.16 (y - 120)$$

$$y - 120 = 4(x - 20)$$

$$x - 20 = 0.16y - 19.2$$

$$y - 120 = 4x - 80$$

$$x - 20 = 0.16y - 19.2$$

$$y = 4x - 80 + 120$$

$$x = 0.16y - 19.2 + 20$$

$$y = 4x + 40$$

$$x = 0.16y + 0.8$$

Advertisement Expenditure 25 Cr.

$$y = 4x + 40$$

$$= 4(25) + 40$$

$$= 100 + 40$$

$$y = 240 \text{ Cr}$$

(04) Calculate

$$\bar{x} = 20 \quad \bar{y} = 12$$

$$s_x = 5 \quad s_y = 25 \quad (\text{standard deviation})$$

$$r = 0.8$$

Reg x on y:

Reg y on x:

$$x - \bar{x} = r \frac{s_x}{s_y} (y - \bar{y})$$

$$y - \bar{y} = r \frac{s_y}{s_x} (x - \bar{x})$$

$$x - 20 = 0.8 \frac{5}{25} (y - 12)$$

$$y - 12 = 0.8 \frac{25}{5} (x - 20)$$

$$x - 20 = 0.16y - 1.92$$

$$y - 12 = 4x - 80$$

$$x = 0.16y - 1.92 + 20$$

$$y = 4x - 80 + 12$$

$$\boxed{x = 0.16y + 18.08}$$

$$\boxed{y = 4x - 68}$$

4. INDEX NUMBER.

- An Index number is a statistical measure to show the changes of any variable such as prices, production and many others with respect to time.

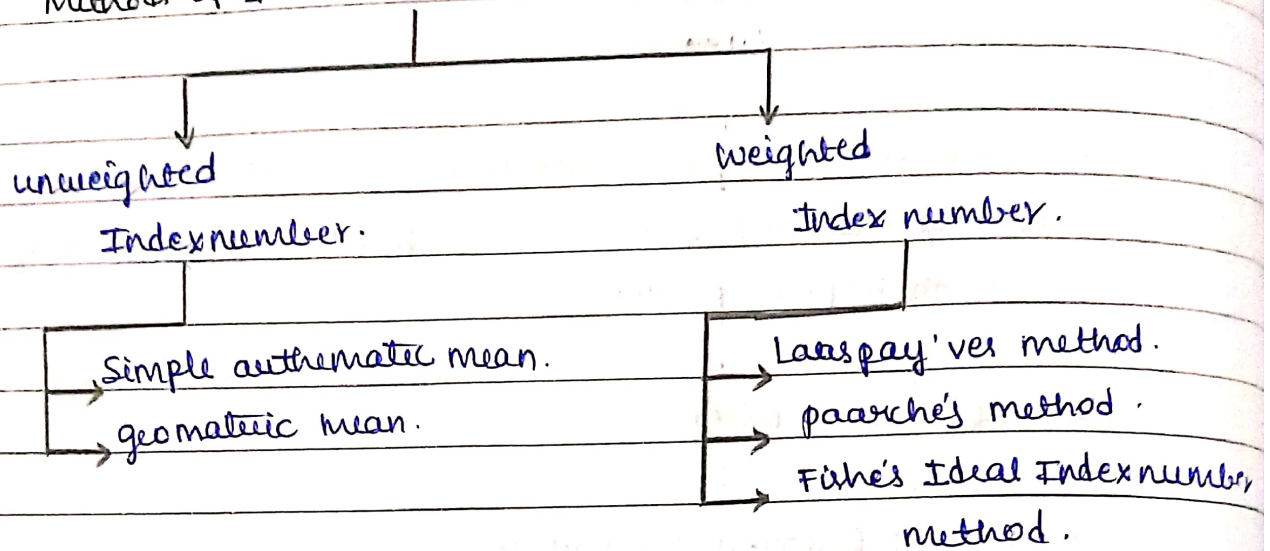
Uses of Index number.

- It helps framing suitable policy
- purchasing power of money.
- Adjustment in wages.
- It helps to comparing living standards.
- It helps to compare predictions.
- It is wider applicability.
- It is useful to deflating.
- It helps in revealing trend

Limitations of Index number.

- It is sampling errors.
 - Quality of the product remains the same.
 - Specific Index for specific purpose.
 - No change in taste, habits and customs.
- ↳ - Variety of methods of construction.

Methods of Index number.



$$\text{Laspeyres method} = P_{01} = \frac{\sum P_1 Q_0}{\sum P_0 Q_0} \times 100$$

$$\text{Paasche's method} = P_{01} = \frac{\sum P_1 Q_1}{\sum P_0 Q_1} \times 100.$$

$$\text{Fisher's Ideal Index method} = P_{01} = \sqrt{\frac{\sum P_1 Q_0}{\sum P_0 Q_0} \times \frac{\sum P_1 Q_1}{\sum P_0 Q_1}} \times 100$$

Basic key points.

Time = 1 and 0.

factors = p and q. (price and quantity)

current year = 1

Base year = 0.

1. Calculate Laspeyres pauché's and Fisher's index numbers.

Items	Current year		Base year	
	price (P ₁)	Quantity (Q ₁)	price (P ₀)	Quantity (Q ₀)
A	30	16	3	15
B	20	18	4	20
C	15	20	6	10
D	8	10	7	5
E	5	9	5	5

$$\text{Laspeyres} = \frac{\sum P_1 Q_0}{\sum P_0 Q_0} \times 100$$

$\sum P_1 Q_0$	$\sum P_0 Q_0$	$\sum P_1 Q_1$	$\sum P_0 Q_1$
450	45	480	48
400	80	360	72
150	60	300	120
40	35	80	70
25	25	45	45
<u>1065</u>	<u>245</u>	<u>1265</u>	<u>355</u>

$$P_{01} = \frac{\sum P_1 Q_0}{\sum P_0 Q_0} \times 100$$

$$= \frac{1065}{245} \times 100 = \boxed{434.69\%}$$

$$\text{pauché's} = P_{01} = \frac{\sum P_1 Q_1}{\sum P_0 Q_1} \times 100$$

$$= \frac{1265}{355} \times 100 = \boxed{356.33\%}$$

$$\begin{aligned}
 \text{Fisher's method } P_01 &= \sqrt{\frac{\sum P_1 Q_0}{\sum P_0 Q_0} \times \frac{\sum P_1 Q_1}{\sum P_0 Q_1}} \times 100 \\
 &= \sqrt{\frac{1065}{245} \times \frac{1265}{355}} \times 100 \\
 &= \sqrt{4.34 \times 3.56} \times 100 \\
 &= \sqrt{15.45} \times 100 \\
 &= 3.93 \times 100 \\
 &= 393\%
 \end{aligned}$$

3. Calculate Fisher's method.

Commodities	Base years.		Current	
	P	Q	P	Q
Rice	4	7	8	8
Sugar	3	6	9	10
Dal	5	5	7	7
Vegetable	2	4	6	5
Kerosene	3	3	2	4

$E P_{100_0}$	$E P_{100_1}$	$E P_{00_0}$	$E P_{00_1}$
56	64	28	32
54	90	18	27 30
35	49	25	35
24	35	8	12
6	8	9	6
<u>175</u>	<u>88</u>	<u>241</u>	<u>119</u>

Fisher's method

$$P_{01} = \sqrt{\frac{\sum P_{100_0}}{\sum P_{00_0}} \times \frac{\sum P_{100_1}}{\sum P_{00_1}}} \times 100$$

$$= \sqrt{\frac{175}{88} \times \frac{241}{119}} \times 100$$

$$= \sqrt{1.988 \times 2.025} \times 100$$

$$= \sqrt{4.0257} \times 100$$

$$= 2.006 \times 100$$

$$\boxed{P_{01} = 200}$$

2. Calculate.

Commodities	2017		2020	
	P ₀	Q ₀	P ₁	Q ₁
A	20	15	10	25
B	18	16	10	15
C	10	18	20	10
D	5	10	15	20
E	4	8	10	30
F	3	7	10	20

$\sum P_1 Q_0$	$\sum P_0 Q_0$	$\sum P_1 Q_1$	$\sum P_0 Q_1$
150	200	300	250
160	180	288	150
360	200	180	200
150	75	50	300
80	40	32	300
70	30	21	200
970	871	1400	1150

$$(A) \text{ Laspeyres } = P_0 I = \frac{\sum P_1 Q_0}{\sum P_0 Q_0} \times 100$$

$$= \frac{970}{871} \times 100 = 111.36\%$$

$$(2) \text{ Paasche } = P_1 I = \frac{\sum P_1 Q_1}{\sum P_0 Q_1} \times 100$$

$$= \frac{1400}{1150} \times 100 = 121.73\%$$

(3) Fisher method.

$$P_{01} = \sqrt{\frac{\sum P_1 Q_0}{\sum P_0 Q_0} \times \frac{\sum P_1 Q_1}{\sum P_0 Q_1}} \times 100$$

$$= \sqrt{\frac{970}{871} \times \frac{1400}{1150}} \times 100$$

$$= \sqrt{1.113 \times 1.217}$$

$$= \sqrt{1.344} \times 100 = 1.163 \times 100$$

$$P_{01} = 116\%$$

4 Calculate Fisher's index number for the following.

Item	Base year		Current year	
	Value	price	Value	price
A	64	16	100	10
B	50	10	81	9
C	49	7	90	10
D	54	9	60	10
E	60	10	30	10

	Q ₀	price (p ₀)	price (p ₁)	Q ₁
A	4	16	10	10
B	5	10	9	9
C	7	7	9	10
D	6	9	6	10
E	6	10	3	10

$\Sigma P_1 Q_0$	$\Sigma P_0 Q_0$	$\Sigma P_1 Q_1$	$\Sigma P_0 Q_1$
40	64	100	160
45	50	81	90
63	49	90	70
36	54	60	90
18	60	30	100
<u>202</u>	<u>277</u>	<u>361</u>	<u>510</u>

Laspeyre's method.

$$P_{01} = \frac{\Sigma P_1 Q_0}{\Sigma P_0 Q_0} \times 100$$

$$= \frac{202}{277} \times 100 = 72.92\%$$

Paasche's method.

$$P_{01} = \frac{\Sigma P_1 Q_1}{\Sigma P_0 Q_1} \times 100$$

$$= \frac{361}{510} \times 100 = 70.78\%$$

Fisher's method.

$$P_{01} = \sqrt{\frac{\Sigma P_1 Q_0}{\Sigma P_0 Q_0} \times \frac{\Sigma P_1 Q_1}{\Sigma P_0 Q_1}} \times 100$$

$$= \sqrt{\frac{202}{277} \times \frac{361}{510}} \times 100$$

$$\begin{aligned}
 &= \sqrt{0.729 \times 0.707 \times 100} \\
 &= \sqrt{0.515 \times 100} \\
 &= 0.717 \times 100 \\
 &= 71.7\%
 \end{aligned}$$

Reversible Test

1. Time Reversible Test.
2. factor Reversal test.

TRT

$$P_{01} = \sqrt{\frac{\sum P_{1Q_0}}{\sum P_{0Q_0}} \times \frac{\sum P_{1Q_1}}{\sum P_{0Q_1}}}$$

$$P_{10} = \sqrt{\frac{\sum P_{0Q_1}}{\sum P_{1Q_1}} \times \frac{\sum P_{0Q_0}}{\sum P_{1Q_0}}}$$

$$P_{01} \times P_{10} = \sqrt{\frac{\sum P_{1Q_0}}{\sum P_{0Q_0}} \times \frac{\sum P_{1Q_1}}{\sum P_{0Q_1}} \times \frac{\sum P_{0Q_1}}{\sum P_{1Q_1}} \times \frac{\sum P_{0Q_0}}{\sum P_{1Q_0}}}$$

$$= \sqrt{1}$$

$$= 1.$$

$$FRT = \frac{\sum P_1 Q_1}{\sum P_0 Q_0}$$

$$P_{01} = \sqrt{\frac{\sum P_1 Q_0}{\sum P_0 Q_0} \times \frac{\sum P_1 Q_1}{\sum P_0 Q_1}}$$

$$Q_{01} = \sqrt{\frac{\sum Q_1 P_0}{\sum Q_0 P_0} \times \frac{\sum Q_1 P_1}{\sum Q_0 P_1}}$$

$$P_{01} \times Q_{01} = \sqrt{\frac{\sum P_1 Q_0}{\sum P_0 Q_0} \times \frac{\sum P_1 Q_1}{\sum P_0 Q_1} \times \frac{\sum Q_1 P_0}{\sum Q_0 P_0} \times \frac{\sum Q_1 P_1}{\sum Q_0 P_1}}$$

$$= \left(\frac{\sum P_1 Q_1}{\sum P_0 Q_0} \right)^2$$

$$= \frac{\sum P_1 Q_1}{\sum P_0 Q_0}$$

1. Calculate Fisher's Index number and also calculate two Reversibility test.

Items	Current		Base year.	
	Price P_1	Qty. Q_1	Price P_0	Qty. Q_0
A	10	9	8	9
B	6	7	8	10
C	7	9	6	10
D	8	5	15	4
E	5	4	7	5

$\Sigma P_1 Q_0$	$\Sigma P_0 Q_0$	$\Sigma P_1 Q_1$	$\Sigma P_0 Q_1$
90	72	90	72
60	80	42	56
70	60	63	54
32	60	40	75
<u>25</u>	<u>35</u>	<u>20</u>	<u>28</u>
277	307	255	285

Fishers Index.

$$P_{01} = \sqrt{\frac{\Sigma P_1 Q_0 \times \Sigma P_1 Q_1}{\Sigma P_0 Q_0 \times \Sigma P_0 Q_1}} \times 100$$

$$= \sqrt{\frac{277 \times 255}{307 \times 285}} \times 100$$

$$= \sqrt{0.902 \times 0.894} \times 100$$

$$= \sqrt{0.806} \times 100$$

$$= 0.897 \times 100$$

$$= 89.7\%$$

TRF.

$$P_{01} \times P_{01} = \sqrt{\frac{\Sigma P_1 Q_0 \times \Sigma P_1 Q_1 \times \Sigma P_0 Q_1 \times \Sigma P_0 Q_0}{\Sigma P_0 Q_0 \times \Sigma P_0 Q_1 \times \Sigma P_1 Q_1 \times \Sigma P_1 Q_0}}$$

$$= \sqrt{\frac{277 \times 255 \times 285 \times 307}{307 \times 285 \times 255 \times 277}}$$

$$= \sqrt{1}$$

$$= \frac{1}{2}$$

$$FRT = P_0 \times Q_0 = \sqrt{\frac{\sum P_1 Q_0}{\sum P_0 Q_0} \times \frac{\sum P_1 Q_1}{\sum P_0 Q_1} \times \frac{\sum Q_1 P_0}{\sum Q_0 P_0} \times \frac{\sum Q_1 P_1}{\sum Q_0 P_1}}$$

$$= \sqrt{\frac{277}{307} \times \frac{255}{285} \times \frac{285}{307} \times \frac{255}{277}}$$

$$= \left(\frac{255}{307} \right)^2$$

$$= \frac{255}{307}$$

2. Calculate Fisher's Index number and also calculate that it satisfy TRI and FRT.

Items	2015		2020	
	P_0	Q_0	P_1	Q_1
A	15	10	30	20
B	10	20	20	17
C	20	30	30	20
D	25	25	25	40
E	16	25	20	16

$\Sigma P_1 Q_0$	$\Sigma P_0 Q_0$	$\Sigma P_1 Q_1$	$\Sigma P_0 Q_1$
300	150	600	300
400	200	340	170
900	600	600	400
625	625	1000	1000
500	400	320	256
<u>2725</u>	<u>1975</u>	<u>2860</u>	<u>2126</u>

Fisher's Index.

$$P_{01} = \sqrt{\frac{\Sigma P_1 Q_0}{\Sigma P_0 Q_0} \times \frac{\Sigma P_1 Q_1}{\Sigma P_0 Q_1}} \times 100$$

$$= \sqrt{\frac{2725}{1975} \times \frac{2860}{2126}} \times 100$$

$$= \sqrt{1.379 \times 1.345} \times 100$$

$$= \sqrt{1.854} \times 100$$

$$= 1.361 \times 100$$

$$= 136.1\%$$

TRT.

$$P_{01} \times P_{10} = \sqrt{\frac{\Sigma P_1 Q_0}{\Sigma P_0 Q_0} \times \frac{\Sigma P_1 Q_1}{\Sigma P_0 Q_1} \times \frac{\Sigma P_0 Q_1}{\Sigma P_1 Q_1} \times \frac{\Sigma P_0 Q_0}{\Sigma P_1 Q_0}}$$

$$= \sqrt{\frac{2725}{1975} \times \frac{2860}{2126} \times \frac{2126}{2860} \times \frac{1975}{2725}}$$

$$= \sqrt{1}$$

$$= \underline{1}$$

FRT.

$$P_{01} \times Q_{01} = \sqrt{\frac{\sum P_{1Q0}}{\sum P_{0Q0}} \times \frac{\sum P_{1Q1}}{\sum P_{0Q1}} \times \frac{\sum Q_{0P0}}{\sum Q_{0P0}} \times \frac{\sum Q_{1P1}}{\sum Q_{0P1}}}$$

$$= \frac{2755}{1975} \times \frac{2860}{2126} \times \frac{2126}{1975} \times \frac{2860}{2755}$$

$$= \left(\sqrt{\frac{2860}{1975}} \right)^2$$

$$= \frac{2860}{1975}$$

3. Calculate Fisher's Index and also its Satisfies Reversibility test.

Items	2020		2022		Q ₁ Quantity
	Value	Price	Value	Price	
A	546	9	10010	101	
B	819	9	10637	9	
C	567	8	4088	6	
D	909	10	546	9	

∑ P_{0Q0}	∑ P_{0Q0}	∑ P_{1Q1}	∑ P_{0Q1}
5400	324	1000	60
5103	729	441	63
2688	392	384	56
4860	810	324	54
18651	2255	2149	233



Fisher Index.

$\Sigma P_1 Q_0$	$\Sigma P_0 Q_0$	$\Sigma P_1 Q_1$	$\Sigma P_0 Q_1$
60	54	100	90
63	81	63	81
56	56	48	48
54	90	54	90
<u>233</u>	<u>281</u>	<u>265</u>	<u>309</u>

Fisher's Index.

$$P_{01} = \sqrt{\frac{\Sigma P_1 Q_0}{\Sigma P_0 Q_0} \times \frac{\Sigma P_1 Q_1}{\Sigma P_0 Q_1}} \times 100$$

$$= \sqrt{\frac{233}{281} \times \frac{265}{309}} \times 100$$

$$= \sqrt{0.829 \times 0.857} \times 100$$

$$= 0.841 \times 100$$

$$= 84.1\%$$

TRF.

$$P_{01} \times P_{10} = \sqrt{\frac{\Sigma P_1 Q_0}{\Sigma P_0 Q_0} \times \frac{\Sigma P_1 Q_1}{\Sigma P_0 Q_1} \times \frac{\Sigma P_0 Q_1}{\Sigma P_1 Q_1} \times \frac{\Sigma P_0 Q_0}{\Sigma P_1 Q_0}}$$

$$= \sqrt{\frac{233}{281} \times \frac{265}{309} \times \frac{309}{265} \times \frac{281}{233}}$$

$$= \sqrt{1}$$

$$= 1$$

FRT.

$$P_{01} \times Q_{01} = \sqrt{\frac{\sum P_1 Q_0}{\sum P_0 Q_0} \times \frac{\sum P_1 Q_1}{\sum P_0 Q_1} \times \frac{\sum Q_1 P_0}{\sum Q_0 P_0} \times \frac{\sum Q_1 P_1}{\sum Q_0 P_1}}$$

$$= \sqrt{\frac{233}{281} \times \frac{265}{309} \times \frac{309}{281} \times \frac{265}{233}}$$

$$= \left(\frac{\sqrt{265}}{281} \right)^2$$

$$= \frac{265}{281}$$

Cost of living Index number / Consumer price Index number

It is Based on Cost of living Index for the families related to their income and Expenditure.

Methods of CPIN [Consumer price Index number]

(1) aggregative Expenditure method.

$$P_{01} = \frac{\sum P_1 Q_0}{\sum P_0 Q_0} \times 100$$

(2) Family Budget Method.

$$P_{01} = \frac{\sum IW}{\sum W}$$

I = Index

W = weight.

1. Calculate CPN by using aggregate Expenditure (family Budget method).

Commodities	Quantity 2005	P_i price in 2008	P_0 price in 2005
Rice	20kg	60	10
sugar	15kg	50	12
Dhal	10kg.	50	15
oil	5l	40	5
veg.	6kg.	30	10.

$\Sigma P_1 Q_0$	(ii) $\Sigma P_0 Q_0$	ΣW	(i) Aggregate Expenditure.	(I) $I = \frac{P_1}{P_0} \times 100$
1200	200	120000	600	
750	180	75000	417	
500	150	50000	333	
200	25	20000	800	
180	60	18000	300	
<u>2830</u>	<u>615</u>	<u>283000</u>	$= \frac{2830}{615} \times 100$	

$P_{01} = 460.16\%$

(2) Family Method.

$$P_{01} = \frac{\Sigma IW}{\Sigma W}$$

$$= \frac{283000}{615}$$

$P_{01} = 460.16\%$

2. Construct the Consumer price index numbers by using aggregate Expenditure and FBM

Commodities	Quantity 2005	P ₁ price 2008	P ₀ price 2005
Rice	25kg	50	10
Sugar	20kg	60	6
Dhal	15kg	60	15
oil	10 oil	30	10
Veg.	10kg	20	5

$\sum P_1 Q_0$	$\sum P_0 Q_0$	$I = \frac{P_1}{P_0} \times 100$	$\sum IW$
1250	250	500	125000
1200	120	1000	120000
900	225	400	90000
300	100	300	30000
200	50	400	20000
<u>3850</u>	<u>745</u>		<u>385000</u>

(1) $CPI = \text{Aggregate Exp.}$

$$P_{01} = \frac{\sum P_1 Q_0}{\sum P_0 Q_0} \times 100$$

$$= \frac{3850}{745} \times 100$$

$$= 516.77$$

(2) Family Budget Method.

$$P_{01} = \frac{\sum IW}{\sum W}$$

= 385000

745

= 516.77%

3. compare the CPI by using aggregate Expenditure method / family budget method.

Commodities	Quantity 2010	P ₀ price 2010	P ₁ price 2012
Nokia	25	500	100
appo	20	400	80
samsung	30	450	90
Lg	25	480	80
poco	35	300	60
vivo	40	350	50
Apple	20	400	80

ΣP ₁ Q ₀	ΣP ₀ Q ₀	$J = \frac{P_1}{P_0} \times 100$	T.W.
2500	12500	20	250000
1600	8000	20	160000
2700	13500	20	270000
2000	12000	16.67	200000
2100	10500	20	210000
2000	14000	14.28	200000
1600	8000	20	160000
<u>14500</u>	<u>78500</u>		<u>1450000</u>

1. Aggregate Expen.

$$P_{01} = \frac{\sum P_1 Q_0}{\sum P_0 Q_0} \times 100$$

$$= \frac{14500}{78500} \times 100$$

$$P_{01} = 18.47\%$$

2. Family budget

$$P_{01} = \frac{\sum IW}{\sum W}$$

$$= \frac{1450000}{78500}$$

$$P_{01} = 18.47\%$$

4. Construct aggregate expenditure / Family Budget.

Commodities	Quantity ^{Q₀} 2010	P ₀ price 2010	P ₁ price 2012
clothes	250	50	500
fuel	300	30	300
vegetables	360	36	360
Dhal	400	40	400
sugar	500	50	500

$\sum P_1 Q_0$	$\sum P_0 Q_0$	$I = \frac{P_1}{P_0} \times 100$	TC
125000	125000	1000	12500000
90000	90000	1000	9000000
129600	129600	1000	12960000
160000	160000	1000	16000000
<u>250000</u>	<u>250000</u>	1000	<u>25000000</u>
754600	231960 75460		75460000

1. Aggregate method.

$$P_{01} = \frac{\sum P_1 Q_0}{\sum P_0 Q_0} \times 100$$

$$= \frac{754600}{231960} \times 100$$

$$= \frac{754600}{231960} \times 100 = 325.31\% = 1000\%$$

2. Family budget

$$P_{01} = \frac{\sum MW}{\sum W}$$

$$= \frac{75460000}{231960}$$

$$= \frac{75460000}{231960} = 325.31\% = 1000\%$$

Missing values.

1. A textile worker earns ₹350 per month. The cost of living index for that particular month is 136. Using the following data find the amt spent by him on house rent/clothing.

group	(W) Expenditure	(I) group Index	WI
food	140	130	25200
clothing	- (x)	150	150x
House Rent	- (y)	100	100y
Fuel	56	110	6160
Miscellaneous	63	86	5040
	$259 + x + y$		$36400 + 150x + 100y$

$$P_{01} = \frac{\sum IW}{\sum W}$$

$$136 = \frac{36400 + 150x + 100y}{350}$$

$$136 \times 350 = 36400 + 150x + 100y$$

$$47600 - 36400 = 150x + 100y$$

$$11250 = 150x + 100y \rightarrow \textcircled{1} \text{ Eqn}$$

$$x + y = 350$$

$$x + y = 259$$

$$x + y = 91 \rightarrow \textcircled{2} \text{ Eqn}$$

$$150x + 100y = 11200$$

$$x + y = 91$$

~~$$x = 91 + y$$~~

$$150(91 + y) + 100y = 11200$$

$$13650 + 150y + 100y = 11200$$

~~$$13650 + 250y = 11200$$~~

$$13650 - 11200 = -250y$$

$$2450 = 250y$$

$$y =$$

multiplied by 150 2nd Eq

$$150x + 150y = 13650$$

$$150x + 150y = 13650$$

$$-50y = -2450$$

$$y = \frac{-2450}{-50}$$

$$\boxed{y = 49}$$

$$x + y = 91$$

$$x + 49 = 91$$

$$x = 91 - 49$$

$$\boxed{x = 42}$$

5. Time Series.

Time series:

An arrangement of statistical data in accordance with time of occurrence in chronological order.

Components of time series:

1. Secular trend.
2. Cyclical trend
3. Seasonal trend.
4. Irregular trend.

Methods of Time series.

1. Method of moving average.
2. Method of least square.

* Formulae of method of least square.

$$Y_c = a + bx$$

Here

Y_c = Year of estimation

$$a = \frac{\sum Y}{n}$$

$$b = \frac{\sum xy}{\sum x^2}$$

(1) calculate method of least square.

5. Time Series..

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- An arrangement of statistical data in accordance with time of occurrence in chronological order.

Components of time series:

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Methods of Time series.

1. Method of moving average.
2. Method of least square.

* problem on method of least square.

$$Y_c = a + bx$$

Here

Y_c = year of estimation

$$a = \frac{\sum y}{n}$$

$$b = \frac{\sum xy}{\sum x^2}$$

(1) calculate method of least square.

(x) years	(y) production (in ton)	$y - \bar{y} = x - \bar{x}$	x^2	$y - \bar{y}$	$(y - \bar{y})^2$
2001	100	-3	9	-300	900
2002	110	-2	4	-200	400
2003	120	-1	1	-100	100
2004 (H)	120	0	0	0	0
2005	135	1	1	135	182.25
2006	130	2	4	260	676
2007	140	3	9	420	1764
	<u>$\Sigma y = 955$</u>		<u>28</u>	<u>-790</u>	<u>3500</u>
				<u>+215</u>	<u>35</u>

$$y_c = a + b \cdot x$$

$$a = \frac{\Sigma y}{n} = \frac{955}{7} = 136.42$$

$$b = \frac{\Sigma xy}{\Sigma x^2} = \frac{35}{28} = 1.25$$

$$y_c = a + b(x)$$

$$136.42 + 1.25(-3) = 132.67$$

$$136.42 + 1.25(-2) = 133.92$$

$$136.42 + 1.25(-1) = 135.17$$

$$136.42 + 1.25(0) = 136.42$$

$$136.42 + 1.25(1) = 137.67$$

$$136.42 + 1.25(2) = 138.92$$

$$136.42 + 1.25(3) = 140.17$$

(b) Straight line best fit the following by the method of least squares.

Year production.

2009 100

2010 100

2011 136

2012 104

2013 112

2014 132

(x)	(y)				
Year	production	$x = x - A$	x^2	xy	$yc = a + b(x)$
2009	100	-3	9	-300	$124.28 + 4.5(-3) = 119$
2010	120	-2	4	-240	$124.28 + 4.5(-2) = 115$
2011	136	-1	1	-136	$124.28 + 4.5(-1) = 120$
2012 (A)	124	0	0	0	$124.28 + 4.5(0) = 124$
2013	118	1	1	118	$124.28 + 4.5(1) = 129$
2014	132	2	4	264	$124.28 + 4.5(2) = 133$
2015	140	3	9	420	$124.28 + 4.5(3) = 138$
	$\Sigma y = 810$		28	-676	
				+802	
				126	

$$a = \frac{\Sigma y}{n}$$

$$= \frac{810}{7} = 124.28$$

$$b = \frac{\Sigma xy}{\Sigma x^2} = \frac{126}{28} = 4.5$$

3. Straight line method for the following data

(x)	(y)				
Year	production	$x = x - A$	x^2	xy	$yc = a + b(x)$
2009	120	-3	9	-360	$151.42 + 3.42(-3) = 141$
2010	150	-2	4	-300	$151.42 + 3.42(-2) = 145$
2011	186	-1	1	-186	$151.42 + 3.42(-1) = 148$
2012 (A)	144	0	0	0	$151.42 + 3.42(0) = 151$
2013	128	1	1	128	$151.42 + 3.42(1) = 155$
2014	182	2	4	364	$151.42 + 3.42(2) = 158$
2015	150	3	9	450	$151.42 + 3.42(3) = 162$
	$\Sigma y = 1060$		28	+942	1060
				-846	
				96	

$$a = \frac{\sum Y}{n} = \frac{1060}{7} = 151.42$$

$$b = \frac{\sum xy}{\sum x^2} = \frac{96}{28} = 3.42$$

4 Estimate the year 2008 fit a straight line method of least square

(x) Year	(y) production	$x = x - \bar{x}$	x^2	xy	$y_c = a + b(x)$
2000	30	-3	9	-90	$39 + 2 \cdot 10(-3) = 38.133$
2001	35	-2	4	-70	$39 + 2 \cdot 10(-2) = 39 + 35$
2002	40	-1	1	-40	$39 + 2 \cdot 10(-1) = 40.137$
2003(A)	42	0	0	0	$39 + 2 \cdot 10(0) = 39$
2004	38	1	1	38	$39 + 2 \cdot 10(1) = 41.41$
2005	43	2	4	86	$39 + 2 \cdot 10(2) = 43$
2006	45	3	9	135	$39 + 2 \cdot 10(3) = 45$
	$\sum y = 273$		28	-200	273
				+259	
				<u>59</u>	

$$a = \frac{\sum Y}{n} = \frac{273}{7} = 39$$

$$b = \frac{\sum xy}{\sum x^2} = \frac{59}{28} = 2.10$$

$$y_c = a + b(x)$$

$$y_{2008} = 39 + 2 \cdot 10(5)$$

$$= 39 + 10 \cdot 50$$

$$= 49.50 \text{ tons}$$

5. straight line trend by using method of least square and estimate the year 2007

(x)	(y)				
Year	production	$x = x - A$	x^2	xy	$Y_c = a + b(x)$
2001	20	-2	4	-40	$30 + 4.5(-2) = 21$
2002	30	-1	1	-30	$30 + 4.5(-1) = 25$
2003(A)	25	0	0	0	$30 + 4.5(0) = 30$
2004	35	1	1	35	$30 + 4.5(1) = 35$
2005	40	2	4	80	$30 + 4.5(2) = 39$
	150		10	-70	150
				+115	
				45	

$$a = \frac{\sum y}{n} = \frac{150}{5}$$

$$= 30$$

$$b = \frac{\sum xy}{\sum x^2} = \frac{45}{10} = 4.5$$

Estimate the year of 2007.

$$Y_c = a + b(x)$$

$$Y_{04} = 30 + 4.5(4)$$

$$= 48$$

6. Estimate the year 2018 in the method of least square

$$Y_c = a + b(x)$$

Year	output	$x = x - A$	x^2	xy	$Y_c = a + b(x)$
2011	15	-3	9	-45	$30 + 5(-3) = 15$
2012	20	-2	4	-40	$30 + 5(-2) = 20$
2013	25	-1	1	-25	$30 + 5(-1) = 25$
2014 (A)	30	0	0	0	$30 + 5(0) = 30$
2015	35	1	1	35	$30 + 5(1) = 35$
2016	40	2	4	80	$30 + 5(2) = 40$
2017	45	3	9	135	$30 + 5(3) = 45$
	$\Sigma Y = 210$		28	-110 +250 <u>140</u>	210

$$a = \frac{\Sigma Y}{n}$$

$$= \frac{210}{7} = 30.$$

$$b = \frac{\Sigma xy}{\Sigma x^2} = \frac{140}{28} = 5.$$

Estimate the year of 2018

$$Y_c = a + b(x)$$

$$Y_{18} = 30 + 5(+8)$$

$$= 30 + 40$$

$$= 70$$

7 Estimate $a + b(x)$ in the year 2020.

Year	output	$x = x - A$	x^2	xy	$y_c = a + b(x)$
2011	5	-4	16	-20	$25 + 5(-4) = 5$
2012	10	-3	9	-30	$25 + 5(-3) = 10$
2013	15	-2	4	-30	$25 + 5(-2) = 15$
2014	20	-1	1	-20	$25 + 5(-1) = 20$
2015(A)	25	0	0	0	$25 + 5(0) = 25$
2016	30	1	1	30	$25 + 5(1) = 30$
2017	35	2	4	70	$25 + 5(2) = 35$
2018	40	3	9	120	$25 + 5(3) = 40$
2019	<u>45</u>	4	<u>16</u>	180	$25 + 5(4) = 45$
	225		<u>60</u>	180 +400 <u>300</u>	225

$$a = \frac{\sum y}{n} = \frac{225}{9} = 25$$

$$b = \frac{\sum xy}{\sum x^2} = \frac{300}{60} = 5$$

Estimate the year 2020

$$y_c = a + b(x)$$

$$y_{20} = 25 + 5(5)$$

$$= 25 + 25$$

$$= \underline{50}$$